



Cambridge O Level

CANDIDATE
NAME

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--



ADDITIONAL MATHEMATICS

4037/13

Paper 1

October/November 2023

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n \{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\begin{aligned} \sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A \end{aligned}$$

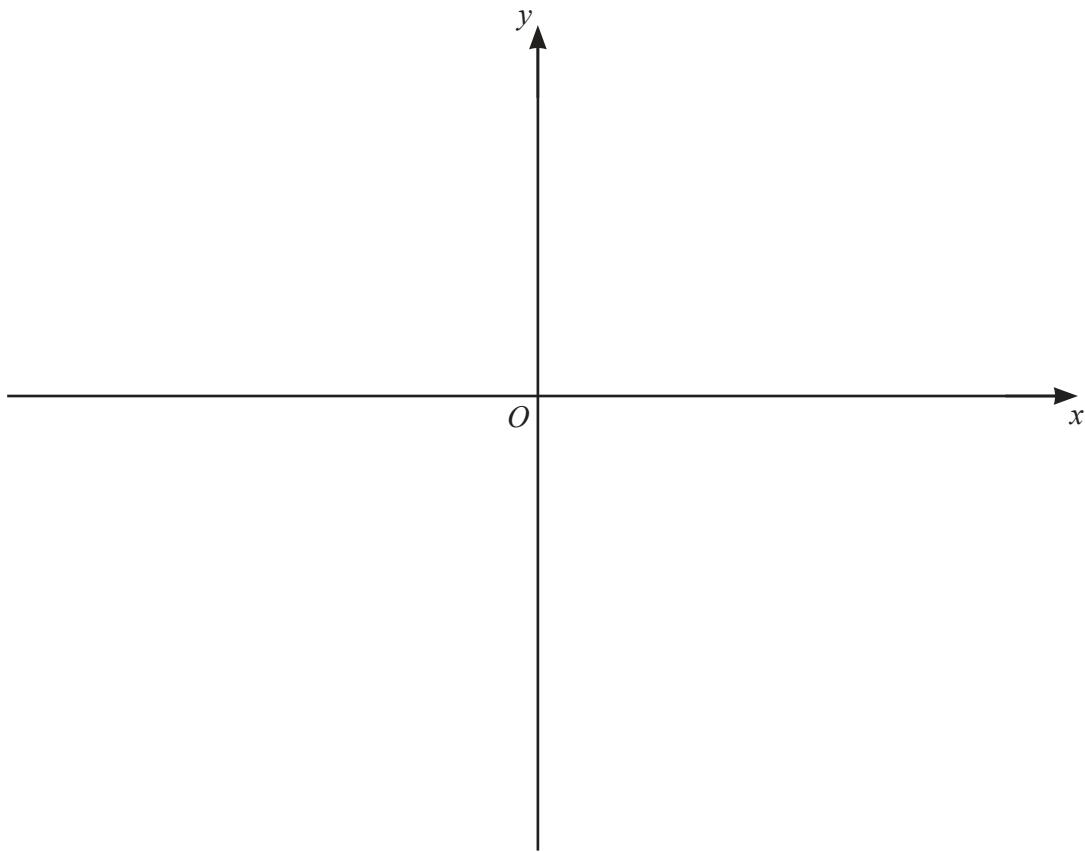
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

1 (a) On the axes, sketch the graphs of $y = 2x + 5$ and $y = |4x - 3|$, stating the intercepts with the coordinate axes. [3]

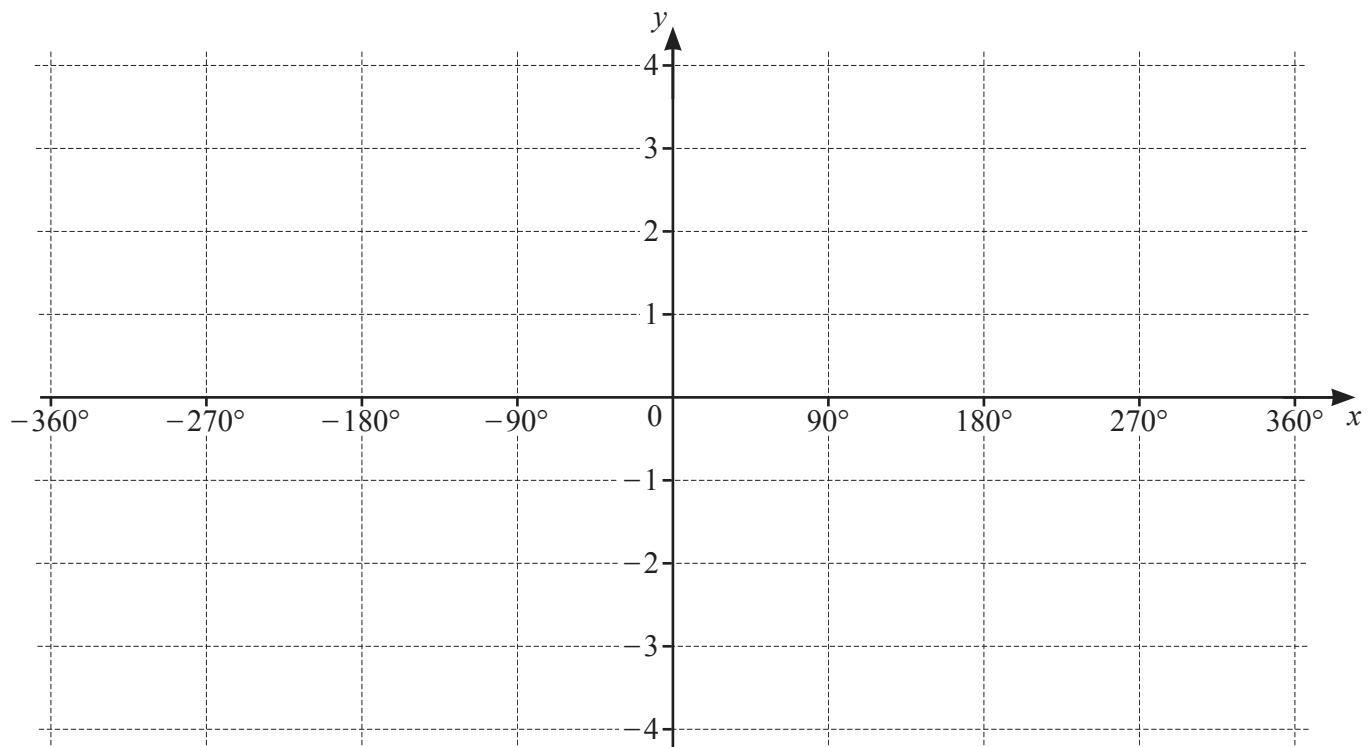


(b) Solve the inequality $|4x - 3| < 2x + 5$. [3]

2 The perpendicular bisector of the line joining the points $\left(-3, \frac{2}{3}\right)$ and $\left(6, -\frac{7}{3}\right)$ passes through the point $(2, k)$. Find the value of k . [4]

3 On the axes, draw the graph of $y = 2 \sin \frac{x}{3} - 1$ for $-360^\circ \leq x \leq 360^\circ$.

[4]



4 The polynomial P is given by $P(x) = ax^3 + bx^2 + 3x + 2$, where a and b are integers. $P(x)$ has a factor of $2x + 1$. $P(x)$ has a remainder of -6 when divided by $x + 1$.

(a) Find the values of a and b . [5]

(b) Show that the equation $P(x) = 0$ has only one real root. [3]

5 (a) A 5-character password is to be formed from the following 10 characters.

Letters	A	B	C	X	Y	Z
Symbols	*	\$	#	&		

No character can be used more than once in any 5-character password.

(i) Find the number of passwords that can be formed. [1]

(ii) Find the number of passwords that can be formed if the password has to contain at least one symbol. [2]

(iii) Find the number of passwords that can be formed if the password has to start with two letters and end with two symbols. [2]

(b) A team of 8 people is to be chosen from 5 doctors, 4 teachers and 6 police officers.

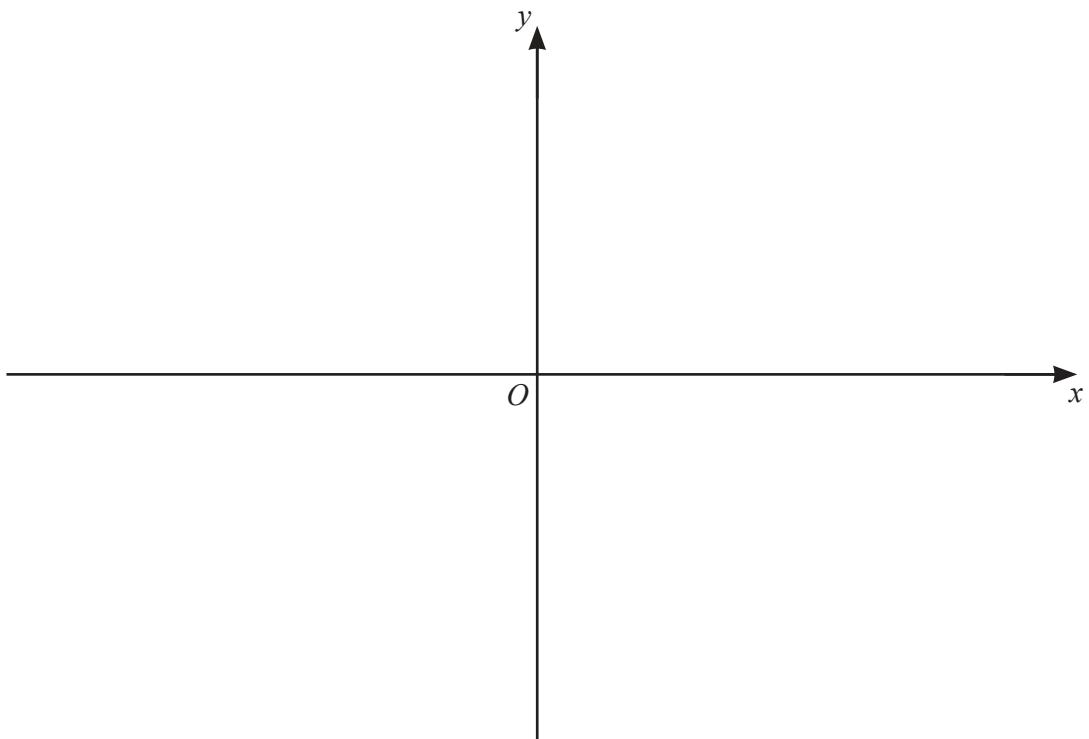
Find how many possible teams have the same number of doctors as teachers. [5]

6 The polynomial $q(x)$ is given by $q(x) = -\frac{1}{3}(2x-1)(x+3)^2$.

(a) Find the x -coordinates of the stationary points on the curve $y = q(x)$.

[4]

(b) On the axes, sketch the graph of $y = q(x)$ stating the intercepts with the coordinate axes. [3]



(c) Find the values of k such that $q(x) = k$ has exactly one solution.

[3]

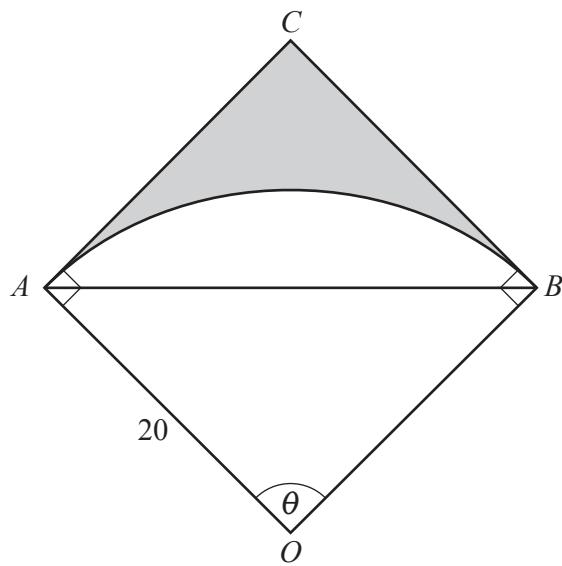
7 Solve the equation $6x^{\frac{1}{3}} - 2x^{-\frac{1}{3}} - 1 = 0$. Give your answers in exact form.

[4]

8 The first three terms, in descending powers of x , in the expansion of $\left(2x^2 - \frac{1}{4x}\right)^n$ can be written in the form $256x^{16} + ax^{13} + bx^c$, where n, a, b and c are integers. Find the values of n, a, b and c . [6]

9 Given that $y = \frac{(5x+2)^{\frac{1}{3}}}{(x-1)^2}$, show that $\frac{dy}{dx}$ can be written in the form $\frac{-(Ax+B)}{3(5x+2)^{\frac{2}{3}}(x-1)^3}$, where A and B are integers. [5]

10 In this question, all lengths are in centimetres and all angles are in radians.



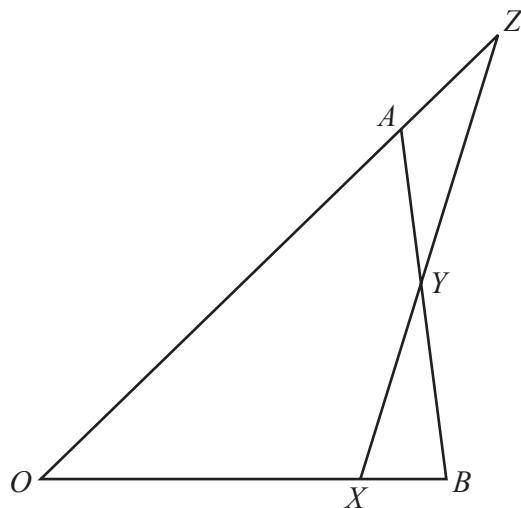
The diagram shows the sector, OAB , of a circle with centre O and radius 20. The perimeter of this sector is 65. The lines CA and CB are both tangents to the circle at the points A and B , so that the triangle ABC is isosceles, with $AC = CB$. The angle AOB is equal to θ .

Find the area of the shaded region.

[9]

Additional working space for question 10.

11



In the triangle OAB , $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

The straight line XYZ is such that:

- $\overrightarrow{OX} = \frac{4}{5}\mathbf{b}$
- $\overrightarrow{AY} = \frac{1}{3}\overrightarrow{AB}$
- $\overrightarrow{AZ} = \mu\mathbf{a}$, where μ is a constant
- $\overrightarrow{YZ} = \lambda\overrightarrow{XY}$, where λ is a constant.

(a) Show that $\overrightarrow{XY} = \frac{2}{3}\mathbf{a} - \frac{7}{15}\mathbf{b}$.

[3]

(b) Find \overrightarrow{YZ} in terms of λ , \mathbf{a} and \mathbf{b} . [1]

(c) Find \overrightarrow{YZ} in terms of μ , \mathbf{a} and \mathbf{b} . [2]

(d) Hence find the values of λ and μ , [3]

Question 12 is printed on the next page.

12 Solve the equation $3 \operatorname{cosec}^2\left(\frac{2x}{3} - \frac{\pi}{3}\right) = 4$, for $0 < x \leq 3\pi$. Give your answers in terms of π . [5]

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.