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4037/13

October/November 2023

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

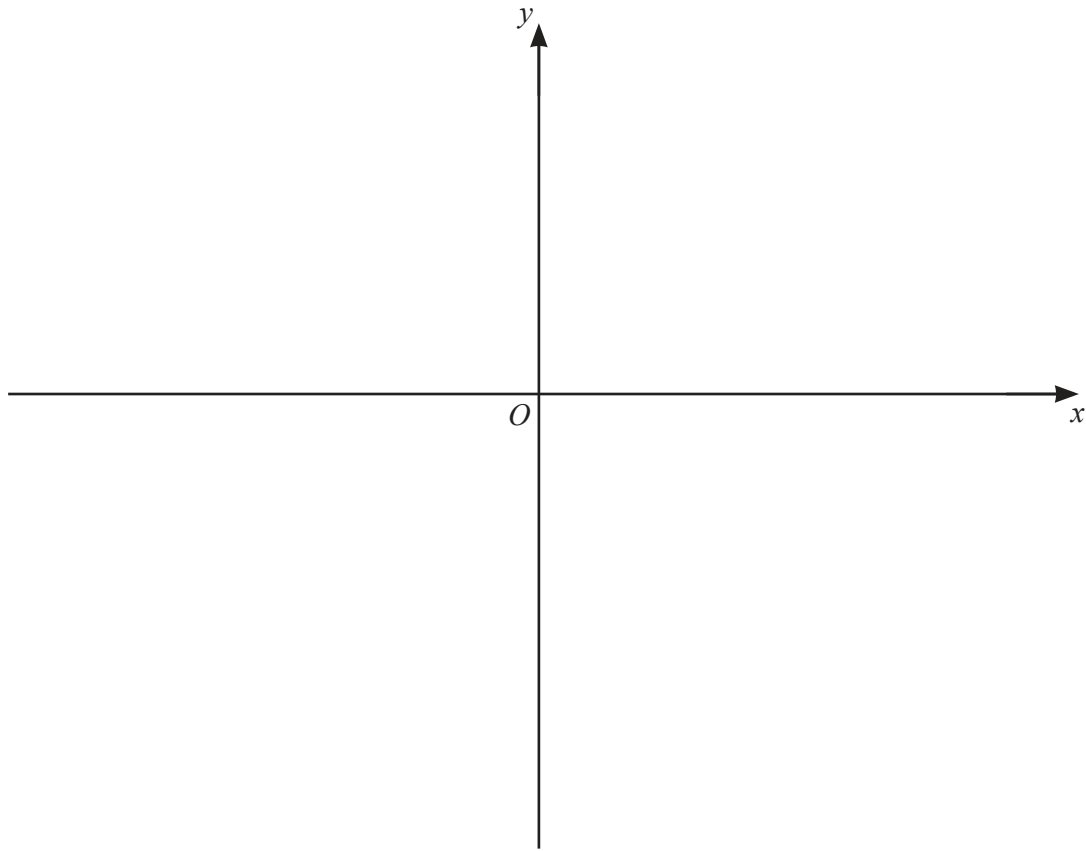
2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 (a) On the axes, sketch the graphs of $y = 2x + 5$ and $y = |4x - 3|$, stating the intercepts with the coordinate axes. [3]

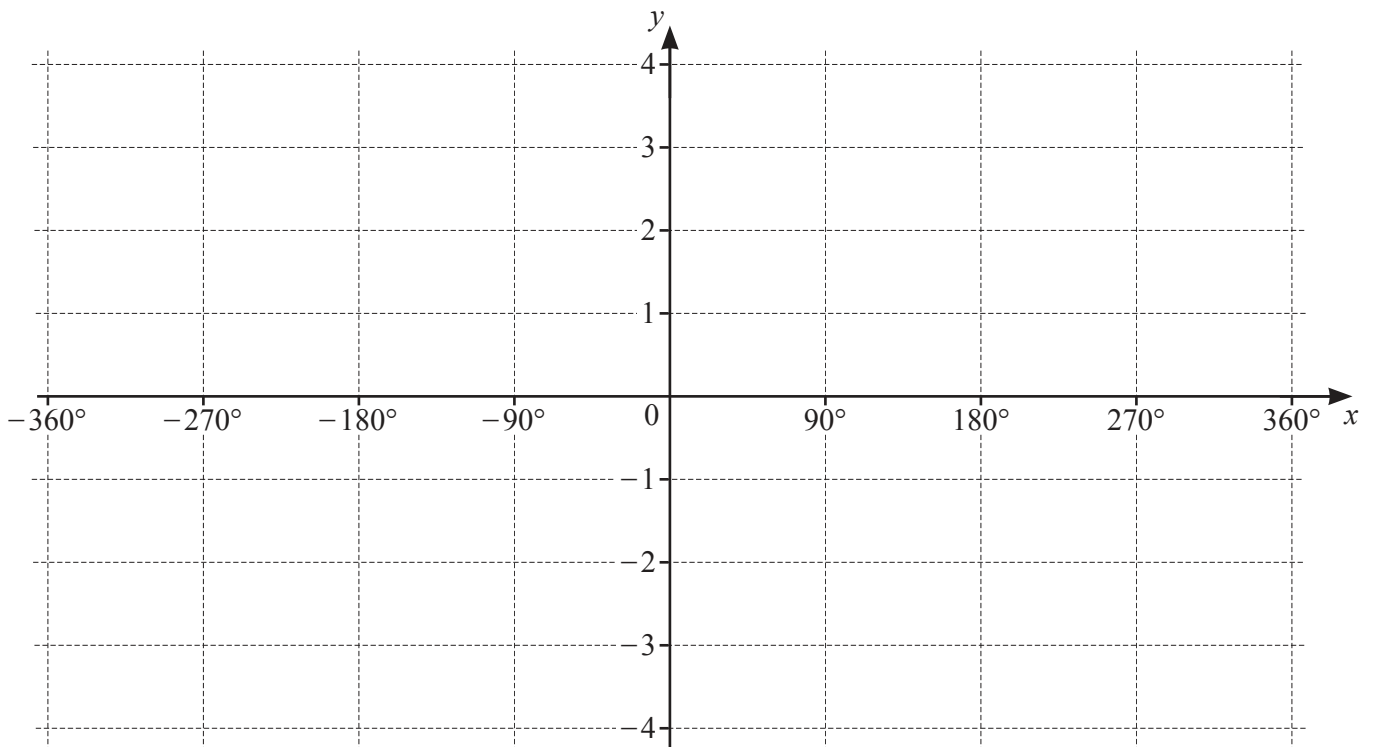


- (b) Solve the inequality $|4x - 3| < 2x + 5$. [3]

- 2 The perpendicular bisector of the line joining the points $\left(-3, \frac{2}{3}\right)$ and $\left(6, -\frac{7}{3}\right)$ passes through the point $(2, k)$. Find the value of k . [4]

3 On the axes, draw the graph of $y = 2 \sin \frac{x}{3} - 1$ for $-360^\circ \leq x \leq 360^\circ$.

[4]



- 4 The polynomial P is given by $P(x) = ax^3 + bx^2 + 3x + 2$, where a and b are integers. $P(x)$ has a factor of $2x + 1$. $P(x)$ has a remainder of -6 when divided by $x + 1$.

(a) Find the values of a and b . [5]

(b) Show that the equation $P(x) = 0$ has only one real root. [3]

- 5 (a)** A 5-character password is to be formed from the following 10 characters.

Letters A B C X Y Z

Symbols * \$ # &

No character can be used more than once in any 5-character password.

- (i)** Find the number of passwords that can be formed. [1]

- (ii)** Find the number of passwords that can be formed if the password has to contain at least one symbol. [2]

- (iii)** Find the number of passwords that can be formed if the password has to start with two letters and end with two symbols. [2]

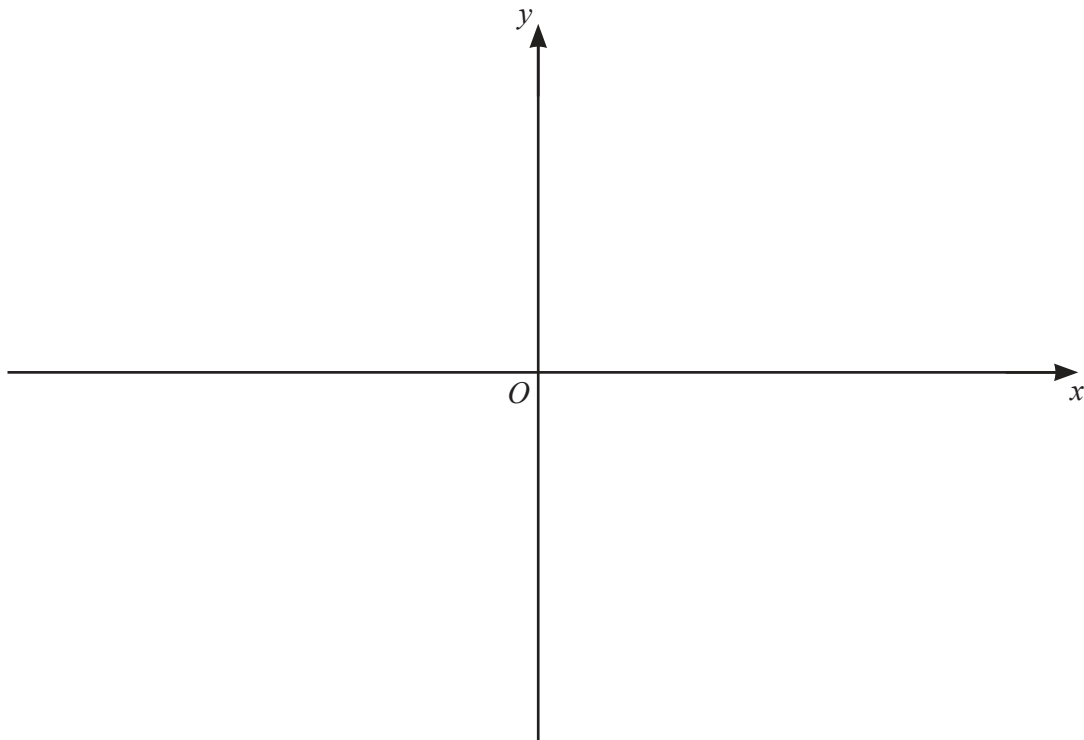
- (b)** A team of 8 people is to be chosen from 5 doctors, 4 teachers and 6 police officers.

Find how many possible teams have the same number of doctors as teachers. [5]

6 The polynomial $q(x)$ is given by $q(x) = -\frac{1}{3}(2x-1)(x+3)^2$.

(a) Find the x -coordinates of the stationary points on the curve $y = q(x)$. [4]

(b) On the axes, sketch the graph of $y = q(x)$ stating the intercepts with the coordinate axes. [3]



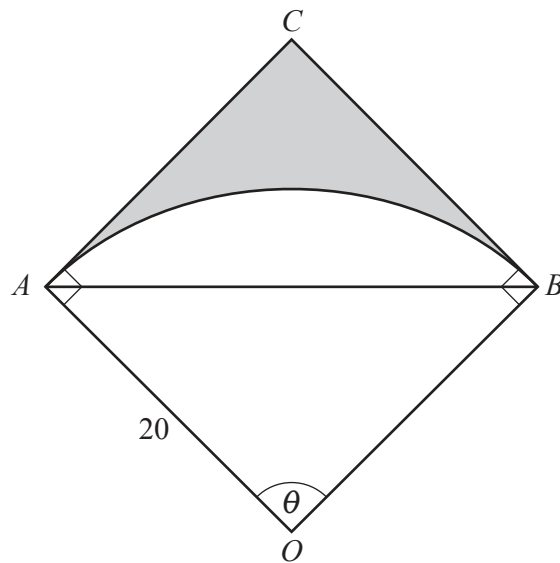
- (c) Find the values of k such that $q(x) = k$ has exactly one solution. [3]

- 7 Solve the equation $6x^{\frac{1}{3}} - 2x^{-\frac{1}{3}} - 1 = 0$. Give your answers in exact form. [4]

- 8 The first three terms, in descending powers of x , in the expansion of $\left(2x^2 - \frac{1}{4x}\right)^n$ can be written in the form $256x^{16} + ax^{13} + bx^c$, where n , a , b and c are integers. Find the values of n , a , b and c . [6]

- 9 Given that $y = \frac{(5x+2)^{\frac{1}{3}}}{(x-1)^2}$, show that $\frac{dy}{dx}$ can be written in the form $\frac{-(Ax+B)}{3(5x+2)^{\frac{2}{3}}(x-1)^3}$, where A and B are integers. [5]

10 In this question, all lengths are in centimetres and all angles are in radians.



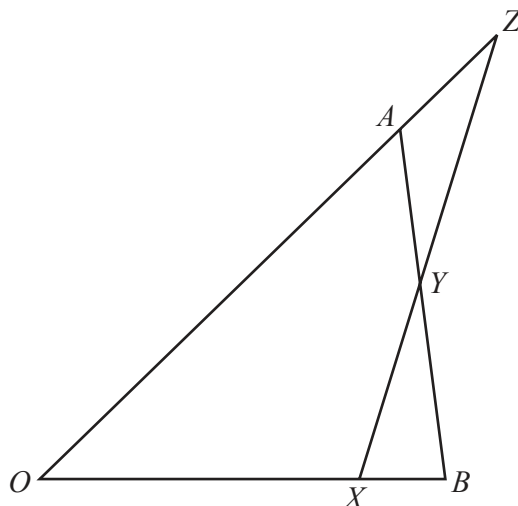
The diagram shows the sector, OAB , of a circle with centre O and radius 20 . The perimeter of this sector is 65 . The lines CA and CB are both tangents to the circle at the points A and B , so that the triangle ABC is isosceles, with $AC = CB$. The angle AOB is equal to θ .

Find the area of the shaded region.

[9]

Additional working space for question 10.

11



In the triangle OAB , $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

The straight line XYZ is such that:

- $\overrightarrow{OX} = \frac{4}{5}\mathbf{b}$
- $\overrightarrow{AY} = \frac{1}{3}\overrightarrow{AB}$
- $\overrightarrow{AZ} = \mu\mathbf{a}$, where μ is a constant
- $\overrightarrow{YZ} = \lambda\overrightarrow{XY}$, where λ is a constant.

(a) Show that $\overrightarrow{XY} = \frac{2}{3}\mathbf{a} - \frac{7}{15}\mathbf{b}$.

[3]

(b) Find \overrightarrow{YZ} in terms of λ , \mathbf{a} and \mathbf{b} . [1]

(c) Find \overrightarrow{YZ} in terms of μ , \mathbf{a} and \mathbf{b} . [2]

(d) Hence find the values of λ and μ , [3]

Question 12 is printed on the next page.

- 12 Solve the equation $3 \operatorname{cosec}^2\left(\frac{2x}{3} - \frac{\pi}{3}\right) = 4$, for $0 < x \leq 3\pi$. Give your answers in terms of π . [5]

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